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# Divergence Thrust Loss Calculations for Convergent-Divergent Nozzles: Extensions to the Classical Case

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# Divergence Thrust Loss Calculations for Convergent-Divergent Nozzles: Extensions to the Classical Case

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## Abstract

This report describes the analytical derivations of the non-axial thrust divergence losses for convergent-divergent nozzles and describes how these calculations are embodied in the Navy/NASA engine computer program. The convergent-divergent geometries considered are: simple classic axisymmetric nozzles, two-dimensional rectangular nozzles, and axisymmetric and two-dimensional plug nozzles. A simple, traditional, inviscid mathematical approach is used to deduce the influence of the ineffectual non-axial thrust as a function of the nozzle exit divergence angle.

## Nomenclature

|                 |  |
|-----------------|--|
| $A_e$           | Nozzle exit area   |
| $A_{th}$        | Nozzle throat area   |
| $C_\theta$      | Divergence thrust loss coefficient                               |
| $F_{G_{Axial}}$ | Gross momentum thrust of a nozzle with finite divergence         |
| $F_{G_{Ideal}}$ | Gross momentum thrust of a nozzle with no divergence             |
| $i$             | Unit vector in axial direction                                   |
| $L$             | Axial distance from nozzle throat to nozzle exit                 |
| $\dot{m}$       | Mass flow rate   |
| $n$             | Unit vector normal to the surface $S$                            |
| $O$             | Origin of flow, specific to nozzle geometry                      |
| $r$             | Radial coordinate  |
| $R$             | Distance from flow origin to nozzle exit                         |
| $S$             | Irregularly-shaped exit surface area specific to nozzle geometry |
| $v_e$           | Nozzle exit velocity vector                                      |
| $v_e$           | Magnitude of nozzle exit velocity vector                         |

|          |  |
|----------|--|
| W        | Width of two dimensional nozzles                               |
| x        | Axial coordinate   |
| $\phi$   | Axial angular coordinate specific to nozzle geometry           |
| $\theta$ | Nozzle exit plane divergence half angle                        |
| $\rho$   | Gas density  |
| $\psi$   | Circumferential angular coordinate specific to nozzle geometry |

## Introduction

Nozzle analysts frequently prefer to account for the many types of nozzle thrust losses by applying correction factors to the ideal performance characteristics of an ideal nozzle. The velocity coefficient, for example, is a factor which, when multiplied by the ideal thermodynamic velocity, accounts for some of the viscous losses in the nozzle. Similarly, correction factors for the non-axial flow of various divergent jets may be analytically derived by expressing the ratio of the momentum of all integrated axial components of the diverging flow across the nozzle exit to the momentum of the flow of an ideal nozzle where all of the exit flow is axial. To aid analysts in non-ideal thrust calculations, we would like to derive geometry-dependent expressions for the nozzle divergence coefficient, defined as

$$C_\theta \equiv \frac{F_{G_{\text{Axial}}}}{F_{G_{\text{Ideal}}}} \quad 1.$$

where  $C_\theta$  is one of possibly several correction coefficients to be applied to the ideal gross momentum thrust. These divergence losses are typically small for most nozzles (on the order of one percent), but may represent a sizeable fraction of the total nozzle performance loss and may be quite significant in nozzles with large degrees of divergence.

The divergence losses of simple axisymmetric nozzles were originally analytically predicted in 1940 by Malina<sup>1</sup>. Currently, computer flow simulations can predict not only these divergence losses but also the divergence losses of geometrically more complex nozzles. Malina's approach, however, is simple and provides a high degree of accuracy. In this paper, Malina's analytical approach is extended to other nozzle designs.

Little data are available on divergence loss analyses for geometrically complex nozzles. For example, Korst and White<sup>2</sup> derive non-axial thrust relations for two dimensional and elliptical supersonic nozzles. The two dimensional nozzle, however, is assumed to have flow emanating from a point source and to have divergent sidewalls--an unlikely configuration for a such a nozzle (See, e.g., Reference 3).

The following is a traditional, mathematical approach using simple vector calculus to calculate the non-axial thrust losses. Uniform, attached, non-interfering inviscid flow is the simplifying assumption for the analysis. Documented in this report are relations for the divergence losses of previously unaddressed nozzles, specifically: the two-dimensional nozzle, the axisymmetric plug nozzle, and the two-dimensional wedge nozzle. The classical axisymmetric supersonic nozzle is the first to be examined. This initial analysis provides the basis and understanding for determining the analogous relations for the other nozzles.

## Analysis

### *The Simple Axisymmetric Nozzle*

Consider the axisymmetric supersonic nozzle shown in Figure 1. Note that the nozzle shown is conical; however, contoured nozzles with flow exiting the nozzle lip at the same angle as the conical nozzle shown may be treated with the following analysis. The axial gross momentum thrust may be expressed as the product of the mass flow rate and the axial exit velocity:

$$F_{G_{\text{Axial}}} = \dot{m} v_{e_{\text{Axial}}} \quad 2.$$

Note that since the familiar pressure thrust term is omitted from Equation 2, this relation represents the momentum component of gross thrust only. This treatment of divergence losses ignores any pressure thrust due to underexpansion, since the divergence loss is an axial momentum loss and must only be applied to the momentum component of the gross thrust.

Assuming flow originating from the source point "O" and non-separating, radial flow of uniform density and velocity across the spherical exit surface "S", the mass flow rate and the axial exit velocity may be written as

$$\dot{m} = \int_S \rho \mathbf{v}_e \cdot \mathbf{n} dS \quad 3.$$

$$v_{e_{\text{Axial}}} = \mathbf{v}_e \cdot \mathbf{i} = v_e \cos \phi \quad 4.$$

where the boldface type denotes a vector quantity. In general, the incremental surface  $dS$  may be expressed as  $r d\psi R d\phi$ . Examining Figure 1, we see that  $dS = R^2 \sin \phi d\psi d\phi$  for this particular nozzle. Substituting

Equations 3 and 4 in Equation 2 and integrating  $\phi$  from zero to  $\theta$  and  $\psi$  circumferentially from zero to  $\pi$ , we arrive at the result

$$F_{G_{\text{Axial}}} = \pi \rho v_e^2 R^2 \sin^2 \theta \quad 5.$$

Similarly manipulating Equation 3,

$$\dot{m} = 2 \pi \rho v_e R^2 (1 - \cos \theta) \quad 6.$$

Combining Equations 1, 5 and 6, and using trigonometric relations, we arrive at the familiar divergence relation for the common simple axisymmetric supersonic nozzle

$$C_\theta = \frac{1 + \cos \theta}{2} \quad 7.$$

If the exit and throat areas are known, and if the divergent walls are not contoured, the divergent half angle may be written as

$$\theta = \arctan \left[ \frac{\sqrt{A_e} - \sqrt{A_{th}}}{L \sqrt{\pi}} \right] \quad 8.$$

where  $L$  is the divergent axial length, defined as the axial distance from the throat plane to the exit plane. This expression is useful if the distance  $L$  is fixed and the nozzle is capable of varying its geometric areas to accommodate changes in nozzle pressure ratio and/or flow conditions.

### *The Two-Dimensional Nozzle*

A supersonic nozzle with a two dimensional, or rectangular, exit is illustrated in Figure 2. Note that a nozzle of this type would most likely have variable geometry divergent walls and fixed parallel sidewalls. Unlike the axisymmetric nozzle above, the hypothetical flow source "O" for this nozzle is linear. For this geometry, the cylindrically-shaped incremental surface area  $dS$  may be expressed as  $W R d\phi$ , where  $W$  is the width of the two-dimensional nozzle. A single integration analysis similar to the procedure above yields the relations for axial gross momentum thrust and mass flow rate

$$F_{G_{\text{Axial}}} = \rho v_e^2 W R (\sin \theta_1 + \sin \theta_2) \quad 9.$$

$$\dot{m} = \rho v_e W R (\theta_1 + \theta_2) \quad 10.$$

Combining these expressions as before leads to the divergence loss coefficient for the two-dimensional nozzle

$$C_\theta = \frac{\sin \theta_1 + \sin \theta_2}{\theta_1 + \theta_2} \quad 11.$$

For unequal exit angles, as in the case of some vectorable-thrust military applications, there may be a very small moment exerted on an aircraft bearing such a nozzle. For the special case of the symmetric two-dimensional nozzle (i.e.,  $\theta_1 = \theta_2 = \theta$ ),

$$C_\theta = \frac{\sin \theta}{\theta} \quad 12.$$

And if the divergent walls are not contoured, the divergent half angle may be written in terms of the nozzle areas as

$$\theta = \arctan \left( \frac{A_e - A_{th}}{2 L W} \right) \quad 13.$$

### *The Axisymmetric Plug Nozzle*

The axisymmetric annular plug nozzle is illustrated in Figure 3. The nozzle half angles along the shroud and along the plug are denoted  $\theta_1$  and  $\theta_2$ , respectively. Unlike the axisymmetric and two-dimensional nozzles above, the hypothetical flow source for this nozzle is circular. An examination of the figure reveals  $dS = R^2 (\sin \theta_2 + \sin \phi) d\psi d\phi$ . The incremental surface area  $dS$  is, in this case, a "dimpled" spherical section. By integrating Equations 2 and 3 over the surface  $dS$ , the expressions for the axial gross momentum thrust and mass flow rate become

$$F_{G_{Axial}} = \pi \rho v_e^2 R^2 (\sin \theta_1 + \sin \theta_2)^2 \quad 14.$$

$$\dot{m} = 2 \pi \rho v_e R^2 [(\theta_1 + \theta_2) \sin \theta_2 + \cos \theta_2 - \cos \theta_1] \quad 15.$$

Thus we may write the divergence coefficient for axisymmetric plug nozzles

$$C_\theta = \frac{\frac{1}{2} (\sin \theta_1 + \sin \theta_2)^2}{(\theta_1 + \theta_2) \sin \theta_2 + \cos \theta_2 - \cos \theta_1} \quad 16.$$



For the special case of the symmetric plug nozzle (i.e.,  $\theta_1 = \theta_2 = \theta$ ),

$$C_\theta = \frac{\sin \theta}{\theta} \quad 17.$$

And if the divergent walls and plug are not contoured,

$$\theta = \arctan \left( \frac{A_e - A_{th}}{2 L \sqrt{\pi A_e}} \right) \quad 18.$$

For the additional special case of the internally cylindrical shrouded plug nozzle (i.e.,  $\theta_1 = 0$  and  $\theta_2 = \theta$ ),

$$C_\theta = \frac{\frac{1}{2} \sin^2 \theta}{\theta \sin \theta + \cos \theta - 1} \quad 19.$$

And if the plug is not contoured,

$$\theta = \arctan \left( \frac{1}{L} \sqrt{\frac{A_e - A_{th}}{\pi}} \right) \quad 20.$$

An example of this geometry is the Low-Angle Plug Nozzle in its cruise orientation (See, e.g., Reference 4.).

Note that for this treatment of plug nozzles, the plug and the shroud exit lip are assumed to be coplanar and the plug and the shroud are both conical. These are not, however, conditions necessary for the accuracy of the divergence coefficient expression. One might envision a contoured shroud and plug which are not coplanar at the exit, but the exit divergence half angles would still describe rays which would converge on some circle "O" in space and would correspond to the conical geometry shown in the figure. Within the limits of the assumptions made, the expressions derived for these divergence coefficients are valid as long as the lip exit half angles are known and the inviscid flow remains uniform and attached at the nozzle exit.

### *The Two-Dimensional Wedge Nozzle*

The two-dimensional wedge nozzle is illustrated in Figure 4. The flow origin in this case is two parallel line sources and the incremental surface area  $dS$  is expressed as  $W R d\phi$ . Performing the integration over the exit plane,

$$F_{G_{Axial}} = 2 \rho v_e^2 W R (\sin \theta_1 + \sin \theta_2) \quad 21.$$

$$\dot{m} = 2 \rho v_e W R (\theta_1 + \theta_2) \quad 22.$$

Combining these as before, the divergence coefficient for the two-dimensional wedge nozzle may be written as

$$C_\theta = \frac{\sin \theta_1 + \sin \theta_2}{\theta_1 + \theta_2} \quad 23.$$

For the special case of the symmetric two-dimensional wedge nozzle (i.e.,  $\theta_1 = \theta_2 = \theta$ ,

$$C_\theta = \frac{\sin \theta}{\theta} \quad 24.$$

And if the divergent walls and wedge are not contoured,

$$\theta = \arctan \left( \frac{A_e - A_{th}}{4 L W} \right) \quad 25.$$

For the additional special case of the internally parallel shrouded wedge nozzle (i.e.,  $\theta_1 = 0$  and  $\theta_2 = \theta$ ), the same result for the divergence loss coefficient occurs:

$$C_\theta = \frac{\sin \theta}{\theta} \quad 26.$$

And if the wedge is not contoured,

$$\theta = \arctan \left( \frac{A_e - A_{th}}{2 L W} \right) \quad 27.$$

An example of this geometry is an early design of the General Electric Low Noise Exhaust Nozzle<sup>5</sup>, prior to the removal of the wedge.

## Results

In Figure 5, the relationship of the divergence loss coefficient to the divergence half angle is illustrated for six nozzle types: simple axisymmetric, symmetric two-dimensional, symmetric plug, plug with cylindrical shroud, symmetric two-dimensional wedge, and two-dimensional wedge with parallel shroud (Equations 7, 12, 17, 19, 24, and 26, respectively). Four of these



nozzles conform to the same divergence rule and are plotted as a single curve. This conformity is a consequence of the mathematics and does not imply similar flow characteristics for the four nozzles.

Figure 6 illustrates the relationship of the divergence loss coefficient to the divergent axial length for the same six nozzle types with throat areas of  $0.05 \text{ m}^2$  and exit areas of  $0.25 \text{ m}^2$  (See Equations 8, 13, 18, 20, 25, and 27.). For plotting the two-dimensional nozzle data in this figure, the nozzle width,  $W$ , is assumed to be identical to the divergent axial length,  $L$ .

It is evident from Figure 5 that simple axisymmetric nozzles exhibit the worst divergence performance for any particular divergence angle. Even plug nozzles with a cylindrical shroud, however, which exhibit the best divergence performance for a given divergence angle, may have appreciable divergence effects. For example, the Low-Angle Plug Nozzle<sup>6</sup>, a cylindrically-shrouded plug nozzle which has a very shallow ten degree plug slope, has divergence effects which account for nearly seventeen percent of its total cruise configuration losses.

### **Divergence Loss Additions to the NNEP89 Computer Program**

These divergence effects are incorporated in the nozzle performance prediction routine of the current (1989) version of the Navy/NASA Engine Program, NNEP89<sup>6</sup>. NNEP89 is a FORTRAN code which descended from both the original Navy/NASA Engine Program, NNEP<sup>7</sup>, and the Navy/NASA Engine Program which can perform chemical equilibrium calculations, NNEPEQ<sup>8</sup>. These very general thermodynamic cycle analysis codes have been used extensively to predict both the design and off-design performance of a variety of Brayton engine cycle configurations ranging from subsonic turboprops to supersonic variable cycle engines. NNEP89's versatility results from the ability of the user to assemble on a piecemeal basis an arbitrary matrix of fans, compressors, turbines, ducts, nozzles, etc., into a complete gas turbine engine model which the code interprets to compute the engine performance characteristics.

The NNEP89 user is offered a choice of six supersonic nozzle geometries for divergence loss calculations. These six nozzles are the obvious choices from the preceding analysis and these nozzles are illustrated in Figure 7. Since it is desirable to keep user inputs to a minimum, nozzles with differing wall angles are not available to the user since another component specification must be used to describe their geometry. For the same reason, the width of all two-dimensional nozzles is assumed to be identical to the divergent axial length. This restriction is, of course, of no consequence if the user inputs the divergent half angle directly. As the nozzle subroutine now stands, three extra specifications are added for supersonic nozzle divergence calculations.

They are: the nozzle divergence half angle, the divergent axial length (which, in addition to the known throat and exit areas, allows the alternate method for the calculation of the divergence half angle), and a geometry type switch. These inputs are described in greater detail in Table 1.

One example of the use of this option in NNEP89 follows: suppose the user is attempting to simulate the performance of a simple convergent-divergent conical nozzle with variable geometry and known dimensions. Further assume that at one geometric position, this nozzle's velocity coefficient (which includes divergence effects) is available from experiment. The user first extracts the effects of this nozzle's divergence loss from the overall velocity coefficient by using the appropriate divergence relation; which, in this particular case, is Equation 7. The user then inserts the resulting velocity coefficient (which now accounts for friction losses alone) into the nozzle input specifications. Then, when the code is executed with the proper divergence inputs, the effects of divergence again are taken into account. The advantage this method offers is that NNEP89 now will approximate the nozzle's divergence losses which correspond to nozzle positions for which no performance data exist.

The input specifications for the new NNEP89 nozzle routine are shown in Table 1. Changes relative to divergence losses are marked in boldface.

## Summary

Since the simplifying assumption of uniform, attached, non-interfering inviscid flow is made, this treatment of divergence losses is somewhat pedagogical. Valuable first-order estimates of these losses can be made in this manner, however, without necessitating complex fluid dynamics computations. These non-axial thrust losses, small in most cases, may represent a sizeable fraction of the total nozzle performance loss and may be quite significant in some nozzle concepts with large degrees of divergence. First-order screenings of candidate nozzle designs may be made using these first-order divergence relations and limits of operational feasibility may also be imposed upon selected nozzle designs.

## References

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3. *New Nozzle Design Aimed at the F-15 and F-16 Aircraft*. Aviation Week and Space Technology, Vol. 117, Sept. 13, 1982.
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5. Majjigi, R.K.: *Preliminary Design Review for NASA Contract NAS3-25414: Low-Noise Exhaust Nozzle Technology Development*. Sept. 24, 1990.
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7. Fishbach, L.H.; and Caddy, M.J.: *NNEP: The Navy/NASA Engine Program*. NASA TM X-71857, 1975.
8. Fishbach, L.H.; and Gordon, S.: *NNEPEQ: Chemical Version of the Navy/NASA Engine Program*. NASA TM 100851, 1988.

**Table 1: NNEP89 Nozzle Input Specifications**

The new divergence loss information is denoted by boldface type. See the NNEP89 user's guide (Reference 6) for more complete information.

### INPUT

The configuration input is defined as follows:

KONFIG(1,N) = 'NOZZ', 4HNOZZ, or 9  
KONFIG(2,N) = Main upstream flow station number  
KONFIG(3,N) = 0  
KONFIG(4,N) = Main downstream flow station number  
KONFIG(5,N) = 0

where N represents the component number. Note there are no secondary flow stations defined for a nozzle.

The nozzle specifications are input as follows:

SPEC(1,N)= Flow area (in<sup>2</sup>); exit for convergent, throat for C-D nozzles. The choked flow area is calculated at the design point.

SPEC(2,N)= Discharge coefficient ( $C_D$ ) or table reference number.

SPEC(3,N)= Nozzle exit static pressure (psia).

SPEC(4,N)= Ambient static pressure (psia). If zero, see SPEC(9,N).

SPEC(5,N)= Velocity coefficient ( $C_V$ ) or table reference number.

SPEC(6,N)= Geometry (nozzle type) Switch:  
=0; Convergent nozzle.  
=1; "Type 1" C-D nozzle. Perfect expansion to ambient static pressure. If SPEC(3,N) > 0, then perfect expansion to SPEC(3,N).  
=2; "Type 2" C-D nozzle. Nozzle exit to throat area ratio is calculated at design. That calculated area ratio is then used for subsequent off-design cases (unless redefined by changing SPEC(8,N)).  
=3; "Type 3" C-D nozzle. Nozzle exit to throat area ratio is supplied by user (see SPEC(8,N)). If this area ratio is less than unity, errors will occur.

**Table 1: NNEP89 Nozzle Input Specifications (Cont.)**

|                    |  |
|--------------------|--|
| <b>SPEC(7,N)=</b>  | <p>Nozzle area (SPEC(1,N)) switch:<br/>         =0; Fix area to input value.<br/>         =1; Vary area in nozzle subroutine to match area required.<br/>         (Note: Even if SPEC(7,N) = 0, nozzle area (SPEC(1,N)) can be varied outside the nozzle routine by using controls or inputting a new value for SPEC(1,N).)</p>  |
| <b>SPEC(8,N)=</b>  | <p>Nozzle exit to throat area ratio. Used at design point and off-design for Type 3 nozzles. Also calculated for Types 1 and 2 nozzles at design point, but only Type 2 nozzles will use this for off-design nozzle calculations. It will be ignored for Type 1 nozzles.</p>   |
| <b>SPEC(9,N)=</b>  | <p>If SPEC(4,N) = 0, set SPEC(9,N) to the component number of the inlet. If SPEC(4,N) and SPEC(9,N) are less than or equal to zero, the program will use SPEC(3,1) (the freestream static pressure of inlet 1) for the nozzle ambient static pressure.</p>   |
| <b>SPEC(10,N)=</b> | <p><b>C-D nozzle exit divergence half angle for divergence loss calculations (deg). This angle may be the exit angle of the shroud, the plug (if any), or both the shroud and plug, depending on the nozzle type switch for divergence loss calculations (see SPEC(12,N)).</b></p>   |
| <b>SPEC(11,N)=</b> | <p><b>C-D nozzle divergent axial length for divergence loss calculations (in). When supplied, the divergence half angle (SPEC(10,N)) is calculated from the known throat and exit areas. This angle will take precedence over any angle specified in SPEC(10,N).</b></p>   |
| <b>SPEC(12,N)=</b> | <p><b>Geometry (nozzle type) switch for C-D nozzle divergence loss calculations:</b><br/>         =0; Disables divergence calculations, divergence loss coefficient set to unity.<br/>         =1; Simple axisymmetric C-D nozzle.<br/>         =2; 2-D C-D nozzle.<br/>         =3; Axisymmetric plug C-D nozzle with equal wall exit half angles on plug and shroud.<br/>         =4; 2-D wedge C-D nozzle with equal wall exit half angles on plug and shroud.<br/>         =5; Axisymmetric plug C-D nozzle with cylindrical (non-divergent) shroud.<br/>         =6; 2-D wedge C-D nozzle with parallel (non divergent) shroud walls.</p> |

**Table 1: NNEP89 Nozzle Input Specifications (Cont.)**

SPEC(13,N)-  
SPEC(15,N)=      Blank

**OUTPUT**

DATOUT(1,N)=      Gross jet thrust (lbf)  
DATOUT(2,N)=      Actual jet velocity (ft/s)  
DATOUT(3,N)=      Total to static pressure ratio at throat  
DATOUT(4,N)=      Nozzle exit area (in<sup>2</sup>)  
DATOUT(5,N)=      Nozzle throat area (in<sup>2</sup>)  
DATOUT(6,N)=      Flow coefficient,  $C_D$   
DATOUT(7,N)=      Velocity coefficient,  $C_v$   
DATOUT(8,N)=      Critical (choked) pressure ratio at throat  
DATOUT(9,N)=      Overall nozzle pressure ratio: nozzle entrance total to  
                         exit static

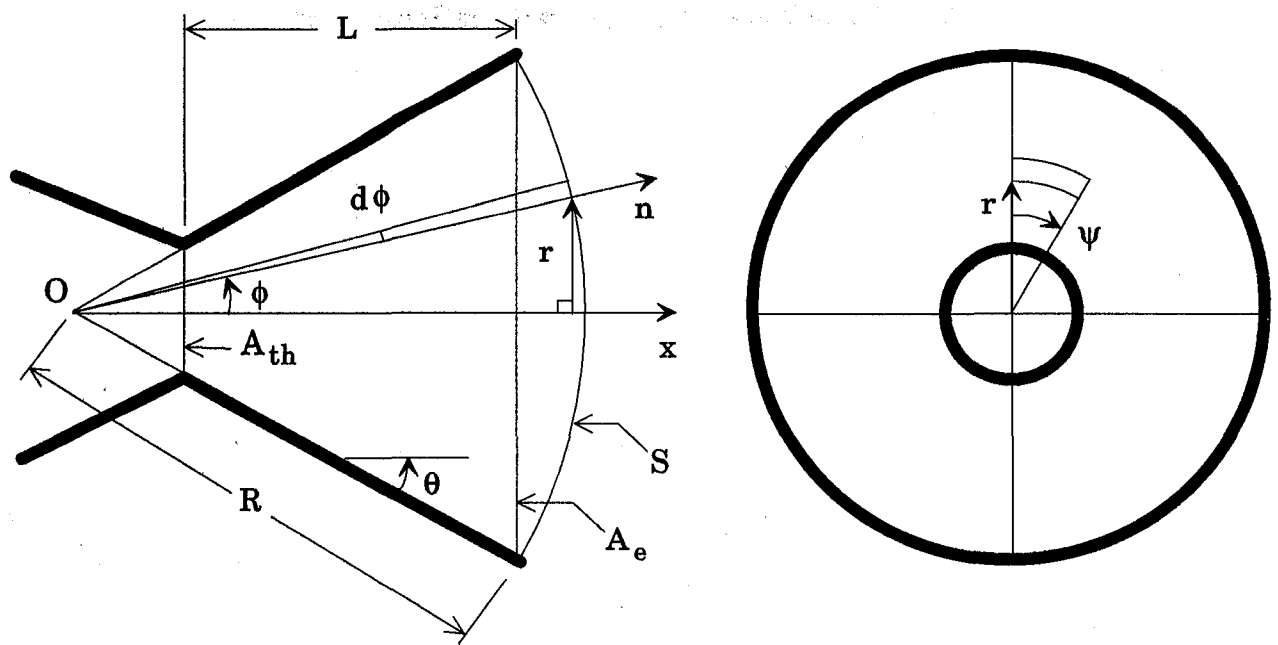


Figure 1: The Axisymmetric Nozzle

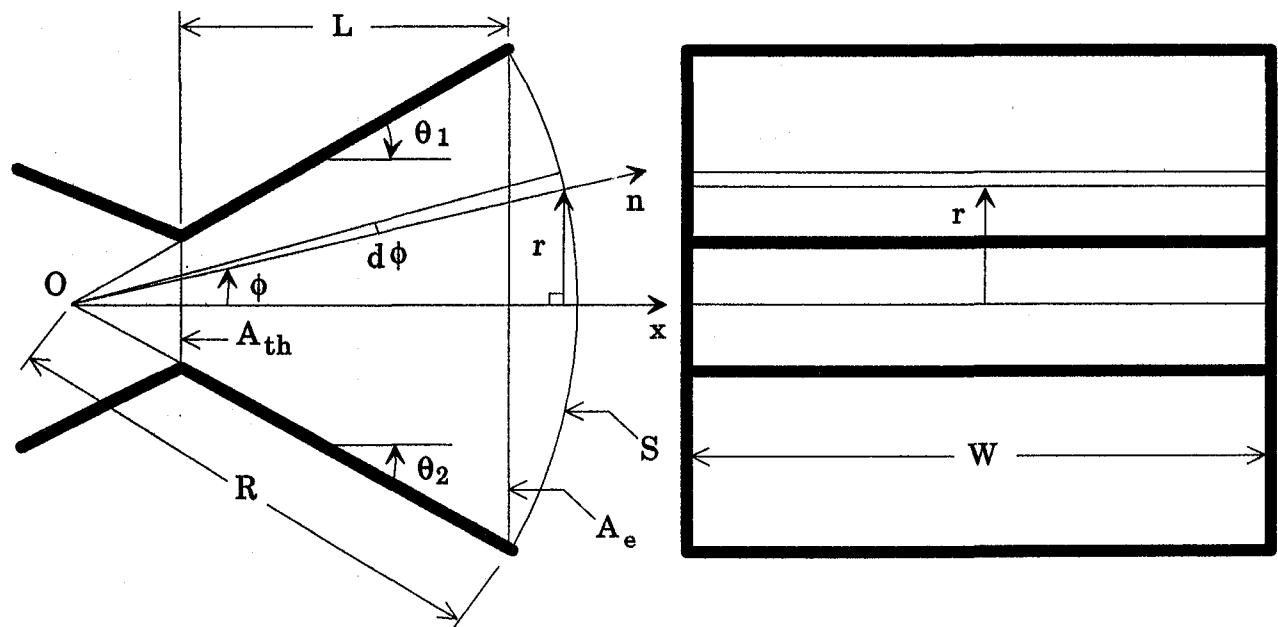


Figure 2: The Two-Dimensional Nozzle



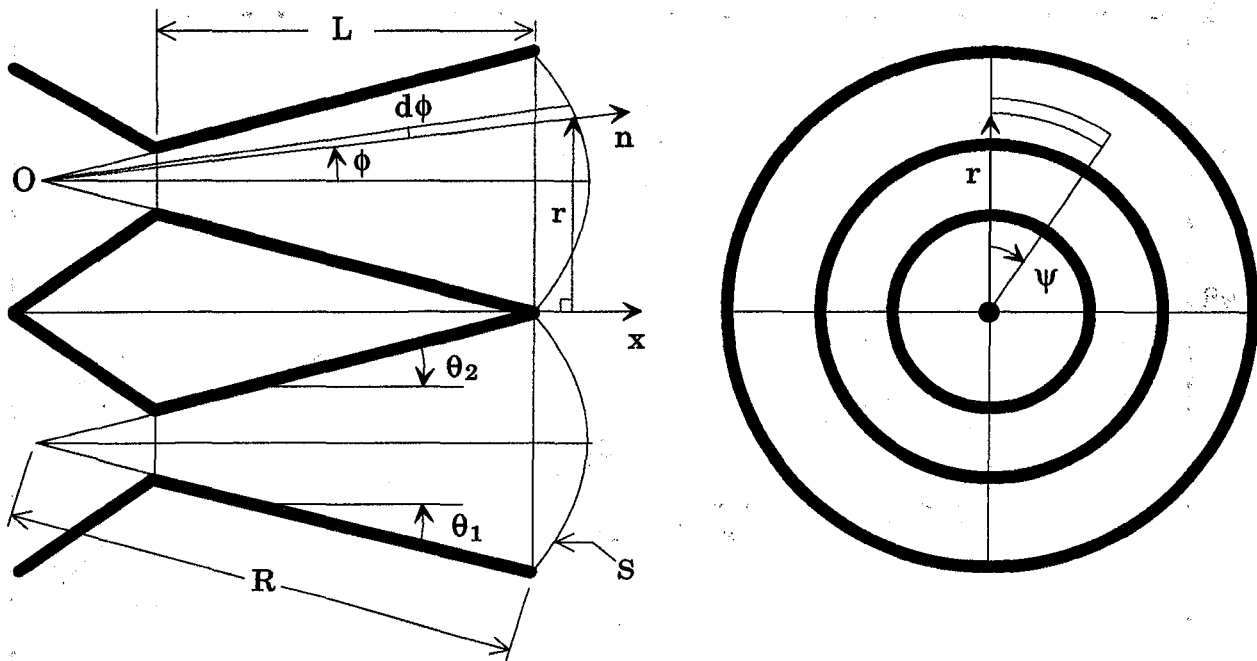


Figure 3: The Axisymmetric Plug Nozzle

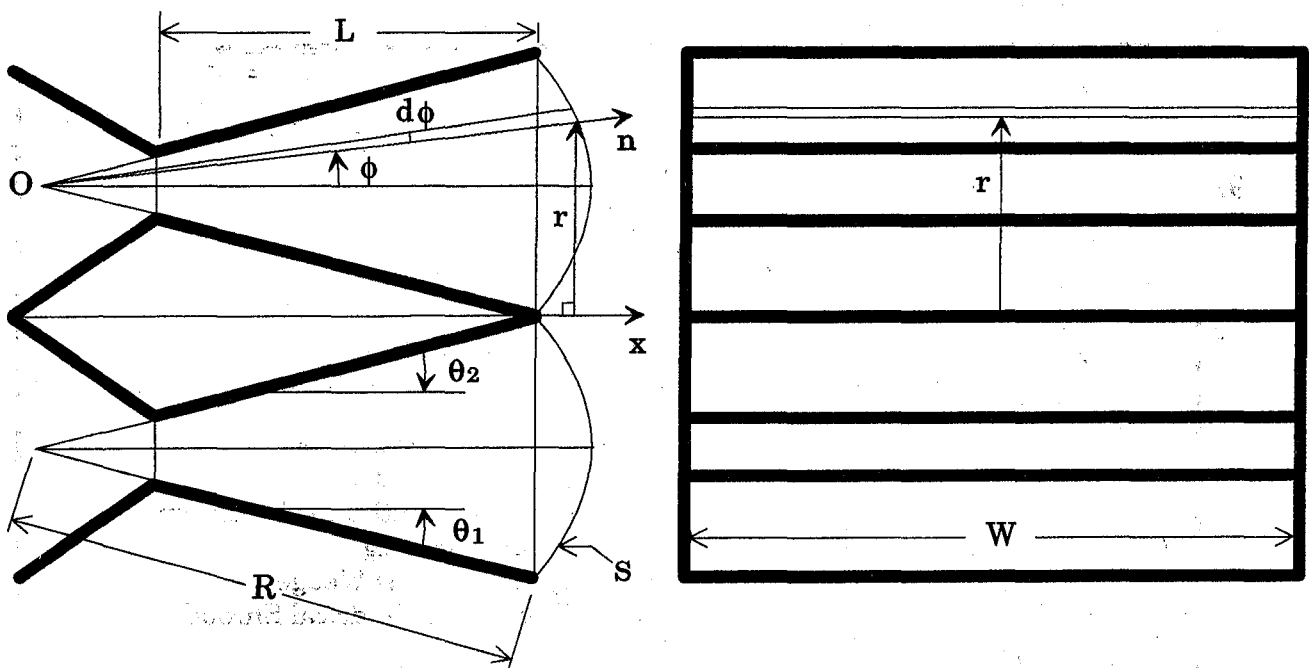


Figure 4: The Two-Dimensional Wedge Nozzle

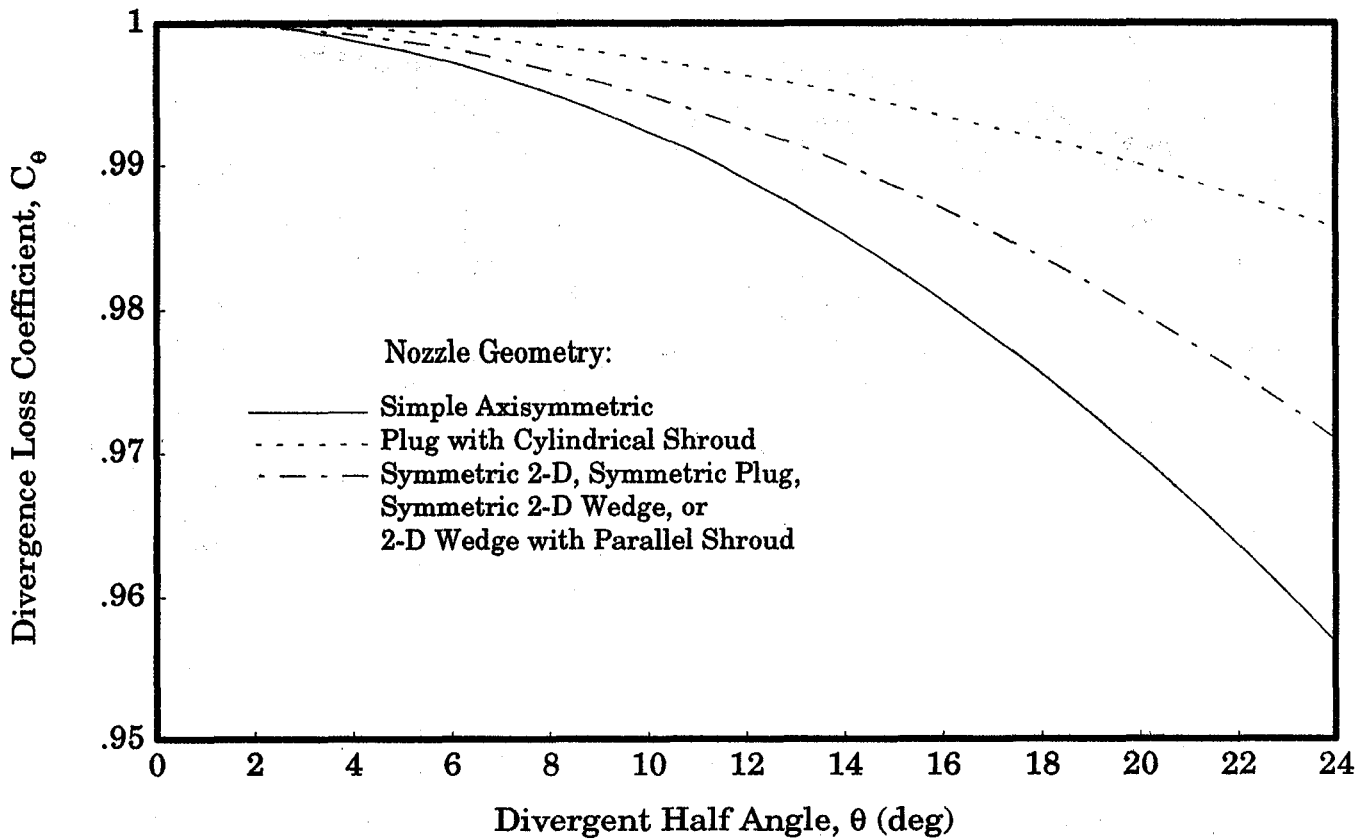


Figure 5: Influence of Divergent Half Angle on  $C_\theta$

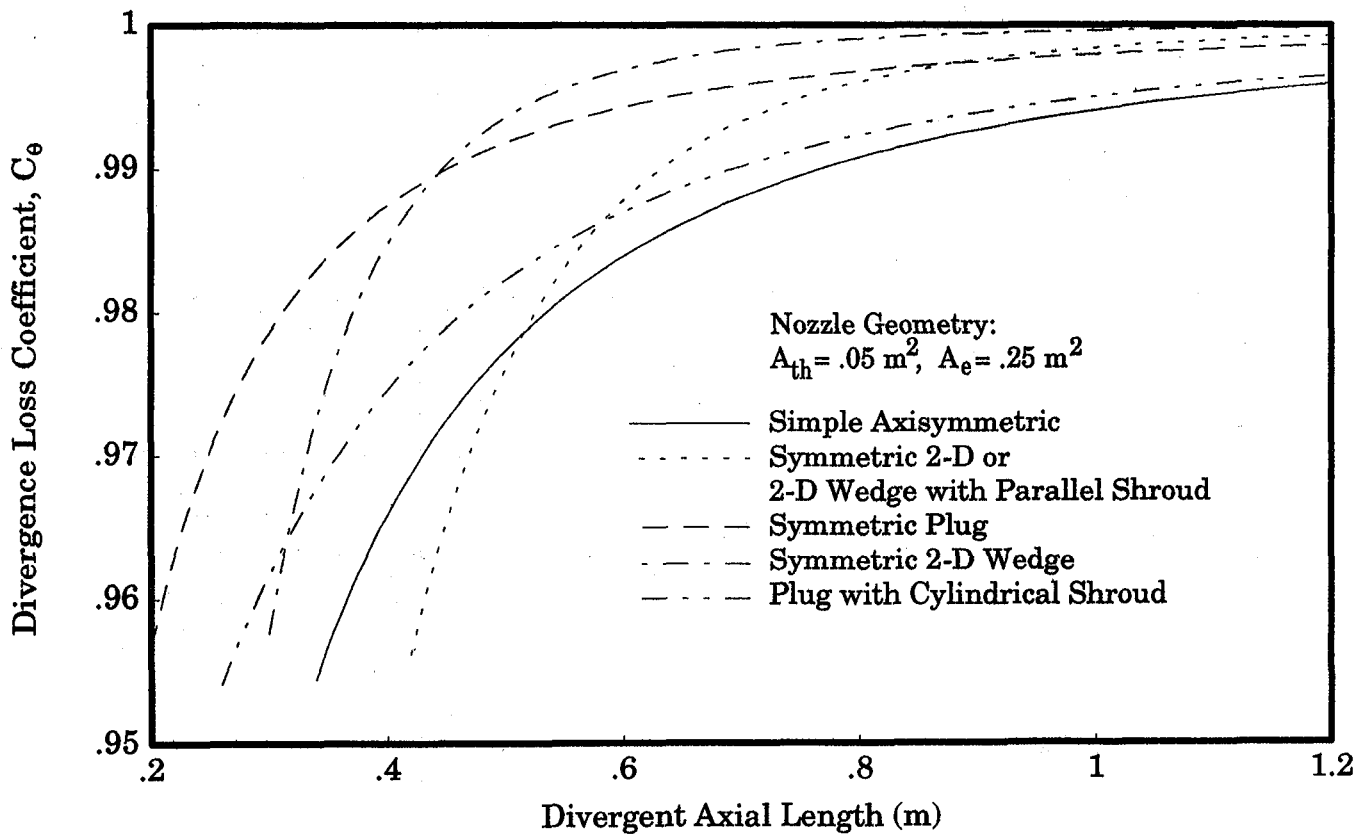
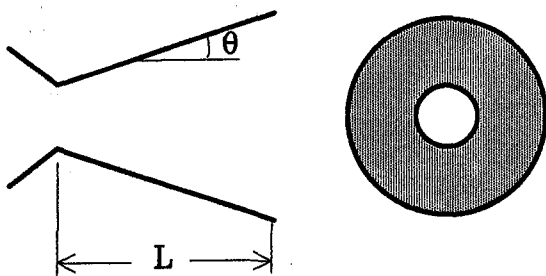
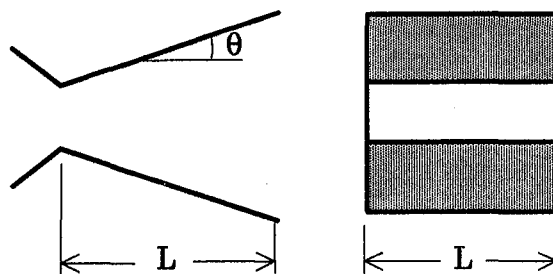


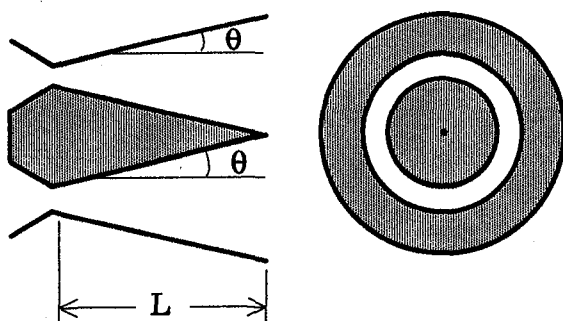
Figure 6: Influence of Divergent Axial Length on  $C_\theta$



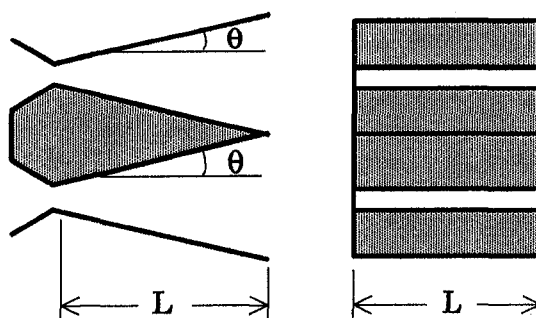
Type 1: Simple Axisymmetric



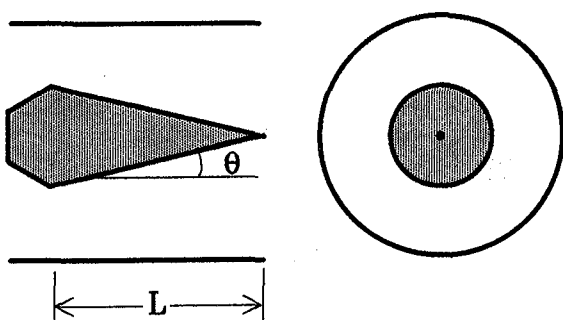
Type 2: Symmetric 2-D



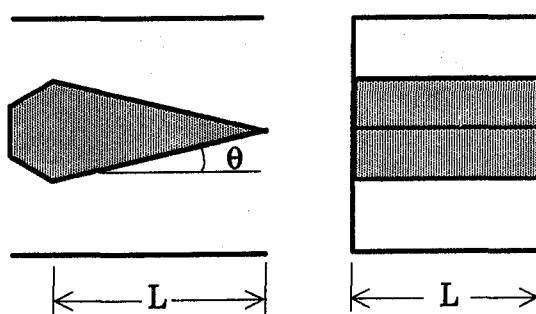
Type 3: Symmetric Plug



Type 4: Symmetric 2-D Wedge



Type 5: Plug with Cylindrical Shroud



Type 6: 2-D Wedge with Parallel Shroud

Figure 7: Nozzle Types for NNEP89 Divergence Calculations

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| 13. ABSTRACT (Maximum 200 words)<br><br>This report describes the analytical derivations of the non-axial thrust divergence losses for convergent-divergent nozzles and describes how these calculations are embodied in the Navy/NASA engine computer program. The convergent-divergent geometries considered are: simple classic axisymmetric nozzles, two-dimensional rectangular nozzles, and axisymmetric and two-dimensional plug nozzles. A simple, traditional, inviscid mathematical approach is used to deduce the influence of the ineffectual non-axial thrust as a function of the nozzle exit divergence angle. |   |  |   |  |
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